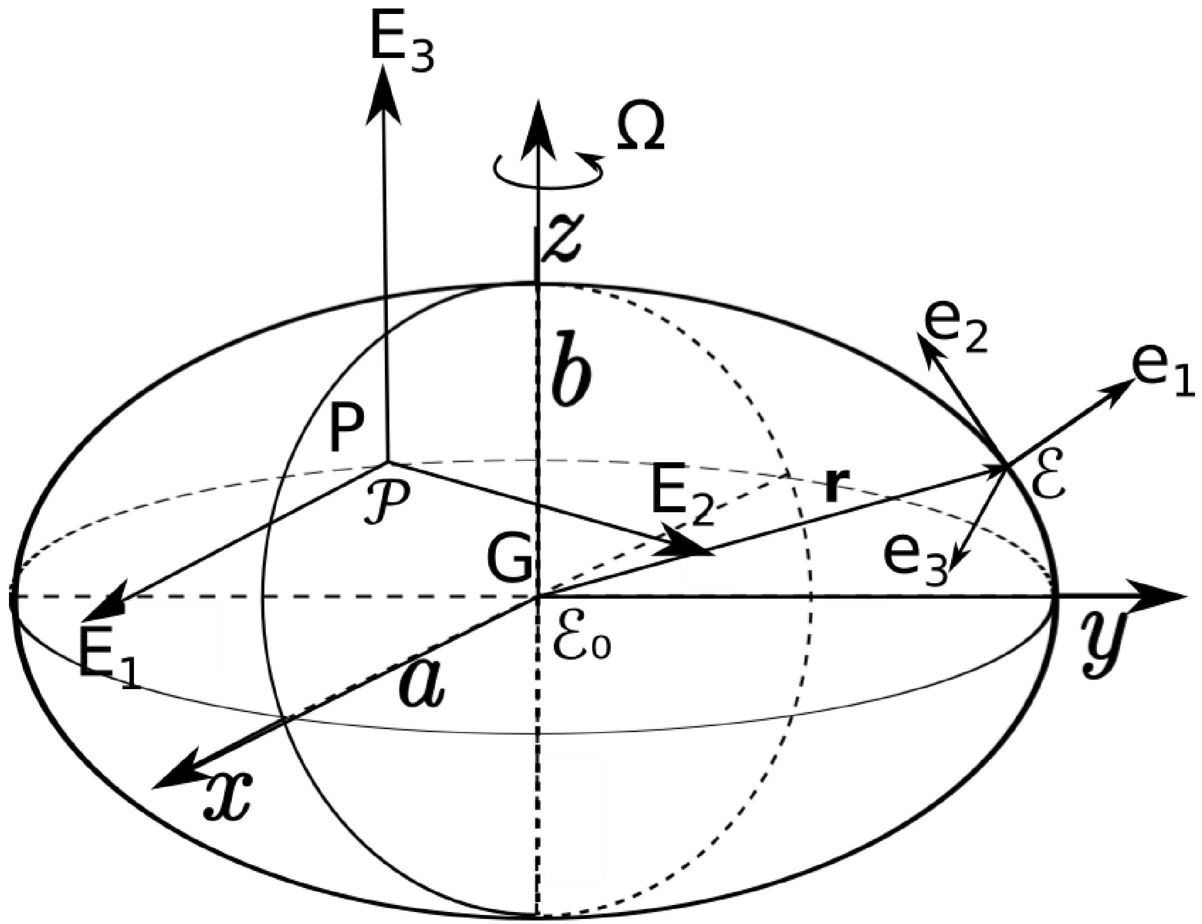
Oblate spheroid

Schematic diagram



The diagram shows an oblate spheroid. The CS $\{\,\boldsymbol{\mathcal{P}}, P, \widehat{\boldsymbol{E}}_i\}$ is the ground fixed. The CS $\{\boldsymbol{\mathcal{E}}_0, G, \boldsymbol{x}_i\}$ is the principal axis of the spheroid. The curvilinear coordinate system is $\{\boldsymbol{\mathcal{E}}, \boldsymbol{r}, \hat{\boldsymbol{e}}_i\}$.

Problem statement

- To solve the incompressible flow of grains on a rotating and gravitating spheroidal body.
- To couple the flow with the dynamics of the spheroid

Immediate questions

1. To conduct multiple axisymmetric landslides on the spheroid
2. To conduct non-axisymmetric landslides
3. To include segregation

Governing Equations

$$\rho\mathring{\boldsymbol{u}} + \boldsymbol{\nabla} \cdot (\rho\boldsymbol{u} \otimes \boldsymbol{u}) = -\boldsymbol{\nabla} \cdot \boldsymbol{P} + \rho(\boldsymbol{b} - \boldsymbol{2\omega} \times \boldsymbol{u} - \boldsymbol{r} \times \boldsymbol{\alpha} - \boldsymbol{a}_G)$$

- $\boldsymbol{u}$ is the velocity of the grains in the CS $\{\boldsymbol{\mathcal{E}}_0, G, \boldsymbol{x}_i\}$ and $\mathring{\boldsymbol{u}}$ is the time derivate in the same CS.
- ρ is the density of the grains and is constant over the domain.
- $\boldsymbol{P}$ is the pressure tensor.
- $\boldsymbol{b} = \boldsymbol{b_0} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$ is the total gravity, where $\boldsymbol{b_0}$ is the gravity due to the spheroidal body, $\boldsymbol{\omega}$ is the spin rate of the body and $\boldsymbol{r}$ is the position of each particle in the principal CS.
- $\boldsymbol{\alpha}$ is the angular acceleration of the spheroidal body.
- $\boldsymbol{a_G}$ is the acceleration of the COM of the spheroid in the ground frame.

Linear Momentum Balance

$$m_c \boldsymbol{r}^{\mathrm{G/P}} + m_R \boldsymbol{r}^{\mathrm{R/P}} = \boldsymbol{0}$$

In other words,

$$m_C \boldsymbol{a}_{\boldsymbol{G}} + m_R \boldsymbol{a}_R = \boldsymbol{0}$$

- m_C is the mass of the core (spheroid) and m_R is the mass of the regolith
- $\boldsymbol{r}^{G/P}$ and $\boldsymbol{r}^{R/P}$ are the position vectors of the COM of spheroid and regolith, respectively, w.r.t to P which is the COM of the system

$$\boldsymbol{r}^P = \frac{m_C \boldsymbol{r}^G + m_R \boldsymbol{r}^R}{m_C + m_R}$$

Angular Momentum Balance

$$\frac{d\boldsymbol{h}_{core}^{P}}{dt} + \frac{d\boldsymbol{h}_{reg}^{P}}{dt} = \mathbf{0}$$

$$\boldsymbol{r}^{G/P} \times m_C \boldsymbol{a}^G + \boldsymbol{\omega} \times \left(\boldsymbol{I}_C^G \cdot \boldsymbol{\omega}\right) + \boldsymbol{I}_C^G \cdot \boldsymbol{\alpha} + \frac{\mathrm{d}}{\mathrm{dt}} \int \rho \boldsymbol{r}^{/P} \times \boldsymbol{U} d\mathcal{V} = 0$$

- $\boldsymbol{h}_{core}^{P}$ is the angular momentum of core about P
- $\boldsymbol{h}_{reg}^{P}$ is the angular momentum of regolith about P
- $\boldsymbol{I}_C^G$ is the moment of inertia of the core about G
- $\boldsymbol{r}^{/P}$ is the position vector of grains w.r.t P
- $\boldsymbol{U}$ is the velocity of the grains in the ground frame

$$\boldsymbol{U} = \boldsymbol{u}_G + \boldsymbol{u} + \boldsymbol{\omega} \times \boldsymbol{r}^G$$

Variable count

- Number of variables

$\boldsymbol{u}$	3
$\boldsymbol{P}$	6
$\boldsymbol{\alpha}$	3
$\boldsymbol{a}_G$	3
Total	15

- Number of equations

Continuity	1
Local LMB of grains	3
LMB of the system	3
AMB of the system	3
Total	10

Angular Momentum Balance

- Simplified using LMB

$$\boxed{\boldsymbol{\omega} \times \left((\boldsymbol{I}_C^G + \boldsymbol{I}_R^G) \cdot \boldsymbol{\omega} + \boldsymbol{h}_{rel}^G \right)} + \boxed{\left(\boldsymbol{I}_C^G + \boldsymbol{I}_R^G \right) \cdot \boldsymbol{\alpha} + \mathring{\boldsymbol{h}}_{rel}^G + \mathring{\boldsymbol{I}}_R^G \cdot \boldsymbol{\omega}} - \boxed{(m_R + m_C)\boldsymbol{r}^G \times \boldsymbol{a}^G} = 0$$

New term due to 3D rotation

Same as Deepayan et al.

New term due to acceleration of the core

- $\boldsymbol{h}_{rel}^G$ is the angular momentum of the grains in the CS $\{\boldsymbol{\mathcal{E}}_0, G, \boldsymbol{x}_i\}$ about the point G.
- $\boldsymbol{I}_R^G$ is the moment of inertia of grains about point G.

System of equations

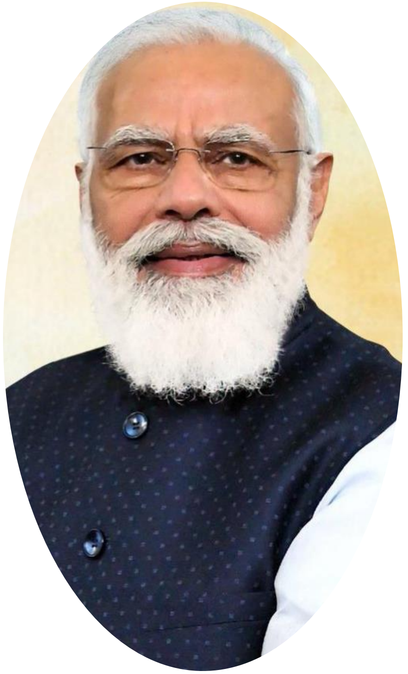
$$\rho\mathring{\boldsymbol{u}} + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{u}\otimes\boldsymbol{u}) = -\boldsymbol{\nabla}\cdot\boldsymbol{P} + \rho(\boldsymbol{b} - \boldsymbol{2\omega}\times\boldsymbol{u} - \boldsymbol{r}\times\boldsymbol{\alpha} - \boldsymbol{a}_G)$$

$$m_C\boldsymbol{a_G} + m_R\boldsymbol{a}_R = \boldsymbol{0}$$

$$\boldsymbol{\omega}\times\left((\boldsymbol{I}_C^G+\boldsymbol{I}_R^G)\cdot\boldsymbol{\omega} + \boldsymbol{h}_{rel}^G\right) + \left(\boldsymbol{I}_C^G + \boldsymbol{I}_R^G\right)\cdot\boldsymbol{\alpha} + \mathring{\boldsymbol{h}}_{rel}^G + \mathring{\boldsymbol{I}}_R^G\cdot\boldsymbol{\omega} - (m_R + m_C)\boldsymbol{r}^G\times\boldsymbol{a}^G = 0$$

आगे देखिये...

- Spheroidal coordinate system
- Basis vectors: are covariant and contravariant basis vectors required?
- Equations in curvilinear coordinates
- Can this system of equations be solved: मोदी है तो मुमकिन हैं



Multiple axisymmetric shallow landslides

Assumptions

- Pure spin
- Core is fixed
- No interaction between shed mass and core

Governing equations:

- $\rho\mathring{\boldsymbol{u}} + \boldsymbol{\nabla} \cdot (\rho\boldsymbol{u} \otimes \boldsymbol{u}) = -\boldsymbol{\nabla} \cdot \boldsymbol{P} + \rho(\boldsymbol{b} - \boldsymbol{2\omega} \times \boldsymbol{u} - \boldsymbol{r} \times \boldsymbol{\alpha})$

- $\left(\boldsymbol{I}_C^G + \boldsymbol{I}_R^G\right) \cdot \boldsymbol{\alpha} + \mathring{\boldsymbol{h}}_{rel}^G + \mathring{\boldsymbol{I}}_R^G \cdot \boldsymbol{\omega} = \boldsymbol{0}$